Climate of the Ocean – Exercise 5

Task 1:

The equations

$$K_m \frac{\partial^2 u}{\partial z^2} + f(v - v_g) = 0$$
$$K_m \frac{\partial^2 v}{\partial z^2} + f(u - u_g) = 0$$

describe the classical Ekman layer in the atmosphere. Solve these equations to determine the height dependence of the departure of the wind field in the boundary layer from geostrophic balance. You can assume that these equations apply throughout the depth of the boundary layer. First, what are the boundary conditions on u and v? You can further assume that the geostrophic wind is independent of height and that the flow is oriented so that the geostrophic wind is in the zonal direction (vg=0). You can see a sketch of the solution in Fig. 1 below.

Hint: Rewrite the equations to arrive with a single equation in the complex velocity (u+iv).



Figure 1: Hodograph of wind components in the Ekman spiral solution. *Arrows* show velocity vectors for several levels in the Ekman layer; the *spiral curve* traces out the velocity variation as a function of height. Points labeled on the spiral show the values of *z*, which is a nondimensional measure of height (Holton, Fig. 8.4).

Task 2:

In the lecture you have derived a solution for the Stommel problem. A variation of this is to use a harmonic viscosity $v\nabla^2 \zeta$ in place of the drag term $-r\zeta$ in the vorticity equation. Then you arrive with the following equation for a square domain of side a:

$$\beta \frac{\partial \psi}{\partial x} = curl_z \tau + v \nabla^2 \zeta = curl_z \tau + v \nabla^4 \psi$$

This is called the Munk problem. Solve this equation as you have done for the Stommel model in the lecture.

Task 3:

Arrange among yourselves to bring laptops for the lecture/exercises on the 4th of January (one laptop between two students will be fine). Only requirements for the laptops are that they can connect to eduroam and have a working internet browser.